University of Cergy-Pontoise **2010-2011**
L1-MI 2$^{\text{nd}}$ semester

Test n° 2

(45 minutes)

Question about course 1. Let E_1 and E_2 be two subspaces of a $\mathbb{R}$-vector space V. For which conditions E_1 and E_2 are supplementary subspaces in V and one can write $V = E_1 \oplus E_2$?

Problem 2. Consider the following polynomial

$$P(X) = X^4 - 4X^3 + 9X^2 - 20X + 20$$

a) Show that 2 is a root of P and determine its multiplicity.

b) **(1)** Show that for every $t \in \mathbb{R}$,

$$P(it) = (t^4 - 9t^2 + 20) + 4it(t^2 - 5)$$

(we recall that $i \in \mathbb{C}$ satisfies $i^2 = -1$)

(2) Deduce that P admits two purely imaginary roots of the form $\pm it_0$ with $t_0 > 0$ which will be specified.

c) By using the previous results, show that

$$P(X) = (X - 2)^2(X^2 + t_0^2)$$

Problem 3. For every $(x, y) \in (]0, +\infty[)^2$, define the following operation :

$$x \star y = x^{\ln(y)} = e^{\ln(x)\ln(y)}$$

a) Show that the operation is respectively

(1) a binary operation on $]0, +\infty[$.

(2) commutative.

(3) associative.

b) Prove that e is the identity element of $\star$.

c) Consider now the set $E =]0, +\infty[- \{1\}$.

(1) Prove that every element $x \in E$ has an inverse element $x^{-1} \in E$ for the operation $\star$ and give the expression of x^{-1}.

(2) $(E, \star)$ forms an algebraic structure. Which one ?