

Continuous assessment n°2

Exercise 1 Let f be the following function :

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}, (x, y) \mapsto \begin{cases} \frac{x^3+3xy^2-y^3}{x^2+y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

1. Show that the partial derivatives of f exist on \mathbb{R}^2 and give their expression.
2. Show that f is not differentiable at $(0, 0)$.

Exercise 2 Let g be the following function :

$$g : \mathbb{R}^2 \rightarrow \mathbb{R}, (x, y) \mapsto \begin{cases} x^2 \ln(x^2 + y^2) & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

1. Show that g is \mathcal{C}^1 on \mathbb{R}^2 .
2. Compute the partial derivatives of order 2 of g when they exist.
3. Show that g is not differentiable at $(0, 0)$.

Exercise 3 Let h be the following function :

$$h : \mathbb{R}^2 \rightarrow \mathbb{R}, (x, y) \mapsto x^2 + 2y^2 + 3y^4 + yx^2$$

1. Determine the local extrema of h .